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Technical Note

Oscillatory enclosed buoyant convection of a fluid with the density maximum

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1. Introduction

Time-dependent buoyant convection in an enclosure, induced by the periodically-varying boundary conditions, has attracted attention in recent years. An important task is to describe the responses of buoyant fluid system to the oscillatory thermal boundary conditions. One example is the air-conditioning system for a room; the thermal load often changes periodically on a daily basis. Of great significance is the existence of resonance. The reports by Lage and Bejan [1] and Antohe and Lage [2], among others, demonstrated that the buoyancy-driven convective system resonates to certain discrete frequencies of the pulsating heat flux at the boundary wall. Under resonance, the amplitude of the fluctuating total heat transfer rate through the vertical mid-plane of the cavity is shown to be maximized. These accounts provided a theoretical estimation of the peak resonance frequency by matching the period of the pulsating heat flux at the wall to the period of the system-wide circulation (flow wheel) of the enclosed fluid. The subsequent experimental efforts [3,4] established that the resonance frequency thus obtained was in order-of-magnitude agreement with the measurements.

In related endeavors, Kwak and Hyun [5] and Kwak et al. [6,7] argued that resonance is expected to occur when the internal gravity waves are excited. Kwak et al. [6,7] ascertained that the resonance frequency calculated by using the basic internal gravity mode was in close agreement with the numerical solutions to the Navier–Stokes equations.

It is noted that all of the previous studies invoked the assumption of a Boussinesq fluid, in which a linear density–temperature relationship is stipulated. However, if the temperature range of the system straddles the temperature T_M at which the density of the fluid reaches the maximum, the conventional Boussinesq-fluid assumption has to be abandoned. For example, for water, the density–temperature relationship in the vicinity of T_M ($\cong 3.98°C$) can not be modeled by a linear function.

In the present note, numerical investigations are made of the response of fluid in a square cavity when the temperature at one vertical wall oscillates about T_M under the non-Boussinesq-fluid approximation. The crux is that, when the wall temperature oscillates about T_M at frequency ω, the corresponding density at the wall oscillates effectively at frequency 2ω, since the density, being maximum at T_M, decreases as the temperature T deviates from T_M both for $T > T_M$ and $T < T_M$. Consequently, it will be shown that resonance is anticipated when the basic internal gravity mode N_i is matched to ω as well as the effective forcing frequency 2ω. This is in contrast to the case of a usual Boussi-

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nesq-fluid in which resonance is seen when N_i is matched to ω.

2. The model and numerical computations

A square cavity, filled with a fluid, is sketched in Fig. 1. The top and bottom horizontal walls are insulated. The temperature T_C at the cold left vertical wall at $X = 0$ is constant. The temperature T_H at the hot right vertical wall at $X = L$ varies sinusoidally in time, $T_H = \overline{T_H} + \Delta T' \sin ft$, in which the cycle-averaged value $\overline{T_H} \equiv T_C + \Delta T$, $\Delta T > 0$, and the amplitude and frequency of oscillation are, respectively, $\Delta T'$ and f. In the present problem setup, $\overline{T_H}$ is equal to the density–maximum temperature T_M. In accordance with the suggestion of Moore and Weiss [8], in the vicinity of T_M, a parabolic function linking density and temperature is selected:

$$\rho = \rho_M\left[1 - \beta(T - T_M)^2\right] \tag{1}$$

In the case of water, the error associated with Eq. (1), with $\beta = 8.0 \times 10^{-6}$ $(°C)^{-2}$, and $T_M = 3.98°C$, is smaller than 4% from 0 to 8°C, and the temperature range of the present problem is supposed to lie within these bounds. The other physical properties of the fluid are taken to be constant at T_M.

The governing time-dependent Navier–Stokes equations incorporating Eq. (1), in properly non-dimensionalized form, were given previously (e.g., in Kwak et al. [9]) and, due to page limitations, they are not reproduced here.

The principal non-dimensional quantities are defined as

$$\theta = \frac{T - T_C}{\bar{T}_H - T_C}, \quad \varepsilon \equiv \frac{\Delta T'}{\bar{T}_H - T_C}, \quad \omega \equiv \frac{f}{N}, \tag{2}$$

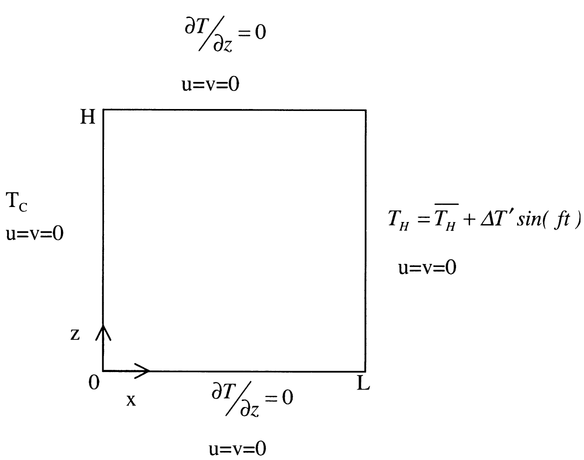


Fig. 1. Flow configuration.

where ε and ω are non-dimensional amplitude and frequency of temperature oscillation at the sidewall, respectively.

The Rayleigh and Prandtl numbers are defined as

$$Ra \equiv \frac{\beta g\left(\bar{T}_H - T_C\right)^2 L^3}{\nu\kappa}; \qquad Pr \equiv \frac{\nu}{\kappa}, \tag{3}$$

where ν and κ are kinematic viscosity and thermal diffusivity, respectively.

It is noted that time is made dimensionless by referring to the Brunt–Vaisala frequency N based on the temperature contrast $(\bar{T}_H - T_C)$, i.e.,

$$N \equiv \left[\frac{\beta g\left(\bar{T}_H - T_C\right)^2}{L}\right]^{1/2} = \frac{(RaPr)^{1/2}\kappa}{L^2} \tag{4}$$

As easily understood, the internal gravity waves are characterized by the prevailing buoyancy, which is depicted by the Brunt–Vaisala frequency, expressed in Eq. (4). Therefore, the nondimensionalization of time by using $1/N$ is a natural choice, and this led to a successful identification of the resonance frequency in [5–7].

The finite volume method was used to secure numerical solutions to the governing equations, following the procedures of SIMPLER algorithm [10]. The QUICK scheme [11] was utilized to discretize the nonlinear advective term. The specifics of the numerical techniques were amply discussed in many prior publications [5]. Most of the calculations were conducted by deploying a mesh network of (62 × 52) staggered grid points in the $(x–z)$ domain. The grid and time-step were varied for repeated calculations of several exemplary flows of Nishimura et al. [12] and Kwak and Hyun [5], and the computed results for Nu were mutually consistent within an accuracy of 1%.

3. Results and discussion

For all the results reported here, $Ra = 10^7$, $Pr = 11.573$, $Ar = 1.0$ to simulate water near 4°C. The range of ω was $0.01 \le \omega \le 10.0$.

To analyze the numerical data, it is advantageous to deploy the following notation:

$$\phi^* = \frac{\phi - \phi_0}{\phi_0}, \tag{5}$$

$$A(\phi) = \frac{\max\{\phi(\tau)\} - \min\{\phi(\tau)\}}{2} \quad \text{for } \tau_0 \le \tau \le \tau_0 + \frac{2\pi}{\omega},$$

In the above, ϕ stands for an arbitrary physical variable, and ϕ_0 denotes the case of the non-oscillatory $(\varepsilon = 0)$ temperature condition at the wall. In Eq. (5),

$A(\phi)$ indicates the amplitude of ϕ for the oscillatory temperature condition at the wall ($\varepsilon \neq 0$).

The total heat transfer across a vertical plane (X = const.) is represented by the Nusselt number Nu, i.e.,

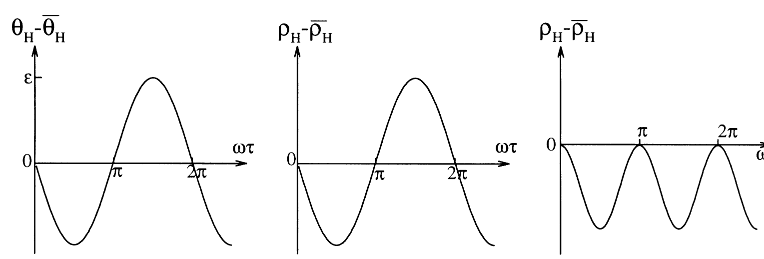
$$N = \int_0^{Ar} \left[U\theta(RaPr)^{1/2} - \frac{\partial \theta}{\partial X} \right] \mathrm{d}Z \tag{6}$$

As remarked earlier, emphasis is placed on the features in the quasi-periodic state; the transitory behavior from $\tau = 0$ to this state is of no direct interest.

The cases of oscillating wall temperature ($\varepsilon \neq 0$) are considered. As the right-wall temperature varies sinusoidally in time with frequency ω, the corresponding density at the right wall changes also in a sinusoidal form with frequency ω (see Fig. 2(b)) for a Boussinesq fluid. However, it is important that, for a non-Boussinesq fluid with a quadratic density–temperature relation, Eq. (1) with $T_{\mathrm{H}} = T_{\mathrm{M}}$, the time-variation of the density at the right wall is periodic with an effective frequency 2ω.

Now, the numerical results for $\varepsilon = 1.0$ are scrutinized. Note that the corresponding density fluctuation at the hot wall is appreciable, i.e., $\varepsilon^2 = 1.0$. A comprehensive series of computations were carried out covering a wide range of ω. Following the procedures adopted in Lage and Bejan [1], the amplitude of fluctuating Nusselt number, $A(Nu^*)$, spatially averaged over the mid-plane $X = 0.5$, is monitored. Fig. 3(a) exemplifies the plot of $A(Nu^*)$ versus ω. Clearly, the results for $A(Nu^*)$ display the primary peak at $\omega_{r1} \cong 0.62$ and the secondary peak at $\omega_{r2} \cong 0.32$. The value of $A(Nu^*)$ at the primary peak is substantially larger than the corresponding steady-state ($\varepsilon = 0$) Nu value. In accordance with the assertion of Lage and Bejan [1], the presence of these peaks of $A(Nu^*)$ manifests the existence of resonance. When $\omega < \omega_{r2}$, $A(Nu^*)$ does not vary much with ω, and $A(Nu^*)$ is approximately 0.80 as ω takes very small values. This implies that, if the wall temperature changes slowly, the system responds as if the fluid is subject to a succession of instantaneous steady-state boundary conditions. In the other limit, as ω exceeds ω_{r1}, $A(Nu^*)$ falls off rapidly, and $A(Nu^*)$ is practically zero when $\omega \geq 4.0$. The impact of high-frequency fluctuations of the boundary conditions is confined to a narrow layer adjacent to the wall, and the oscillatory nature is not felt in much of the interior region. These qualitative observations on the influence of the frequency of wall-temperature pulsation are consistent with the earlier findings for a Boussinesq fluid [5–7].

The time-histories of Nu^* at the mid-plane $X = 0.5$ are exemplified for four different values of ω in Fig. 3(b). As expected, when ω is large (see the curve for $\omega = 1.00$), the pulsating part of heat transfer in the interior diminishes. When ω is at the primary resonance frequency ($\omega = \omega_{r1} \cong 0.62$), the pulsation of interior heat transfer proceeds at frequency ω with a substantially magnified amplitude. The value $A(Nu^*)$ is lower at the secondary resonance frequency ($\omega = \omega_{r2} \cong 0.32$) than at $\omega = \omega_{r1}$. At an even lower ω (see the curve for $\omega = 0.10$), $A(Nu^*)$ settles to a value which is lower than that at $\omega = \omega_{r2}$. It is seen that, at low pulsation frequencies, the fluctuating $Nu^*]_{X=0.5}$-curves contain the component of frequency 2ω in addition to the primary component of frequency ω. It is recalled that, under the quadratic density–temperature relation of Eq. (1), a wall-temperature pulsation at frequency ω brings forth a wall-density oscillation at frequency 2ω. The pulsation of $Nu^*]_{X=0.5}$ at 2ω is indicative of the afore-said density-fluctuation at 2ω. The influence of 2ω-components is meager in the case of moderate- and high-frequency oscillations (see the curves of $\omega = 1.00$ and $\omega = 0.62$ of Fig. 3(b)). As stressed earlier, the effect of high-frequency pulsations of the boundary conditions can not penetrate into the bulk of the interior. Note that, for ω moderate or large, such as $\omega = 0.62$ or $\omega = 1.00$, 2ω falls in the high-frequency regime in the present range of parameters.



(a) (b) (c)

Fig. 2. Periodic behavior of the pulsating conditions at the wall (X = 1.0). (a) Temperature, (b) density for a Boussinesq fluid, (c) density for a non-Boussinesq fluid of Eq. (1).

Kwak and Hyun [5] argued that resonance occurs when the system resonates to the characteristic frequency. For the buoyant enclosed fluid, this frequency is identified to be the basic mode of internal gravity oscillations which are supported by the prevailing density-stratification in the interior. To the lowest-order approximation, the interior fluid can be assumed inviscid for large *Ra*. It follows that the frequencies of internal gravity oscillations can be calculated by estimating the vertical gradient of density in the interior [13]. Specifically, for a square cavity, Kwak and Hyun [5] and Kwak et al. [6,7] adopted a formula for the basic mode N_1 of internal gravity oscillation [13]:

$$N_1 = \frac{C_i}{\sqrt{2}}. \quad (7)$$

In the above, C_i represents the mean vertical gradient of density in the interior region. For the parameter set of Fig. 3, a curve-fitting to the computed data of average values of density gradient at $X = 0.5$ yields $C_i \cong 0.8706$, which, upon substitution into Eq. (7), leads to $N_1 \cong 0.62$. Resonance is expected when the pulsation frequency ω matches N_1, which points to the primary resonance frequency $\omega = N_1$. This prediction is in close agreement with the numerically-determined value $\omega_{r1} \cong 0.62$. It is emphasized that the presence of the secondary resonance frequency is a unique feature of a non-Boussinesq fluid of Eq. (1). As explicitly stated, due to the quadratic nature of temperature–density relationship, the wall-temperature pulsation at ω causes the wall-density fluctuation at 2ω as well as at ω. Consequently, resonance is also anticipated when the effective frequency 2ω matches N_1, i.e.,$\omega = N_1/2 \cong 0.31$. This predicted value of secondary resonance frequency is in broad agreement with the numerical result of $\omega_{r2} \cong 0.32$. Minor discrepancies are attributed to the uncertainties in numerical fitting and to the fact that the system is not strictly inviscid, among others. These exercises provide further justification to the above physical interpretation in predicting the resonance frequencies based on the notion of internal gravity oscillations. Also, the existence of the secondary resonance frequency is identified to be a manifestation of the effect of a non-Boussinesq fluid.

4. Conclusion

Comprehensive numerical results demonstrate the existence of resonance in the present buoyant convective system. The primary resonance frequency ω_{r1} is shown to match the basic mode N_1 of internal gravity oscillations. The presence of secondary resonance frequency $\omega_{r2}[= N_1/2]$ is peculiar to the non-Boussinesq fluid. Due to the quadratic density–temperature(ρ–T) relationship, the effective pulsation frequency for density at the wall contains 2ω in addition to ω. The above physical interpretations are supportive of the argument of Kwak and Hyun [5]. It has been asserted that resonance is expected when the fundamental characteristic frequency of the system is excited, which, in this case, is identified to be the internal gravity mode.

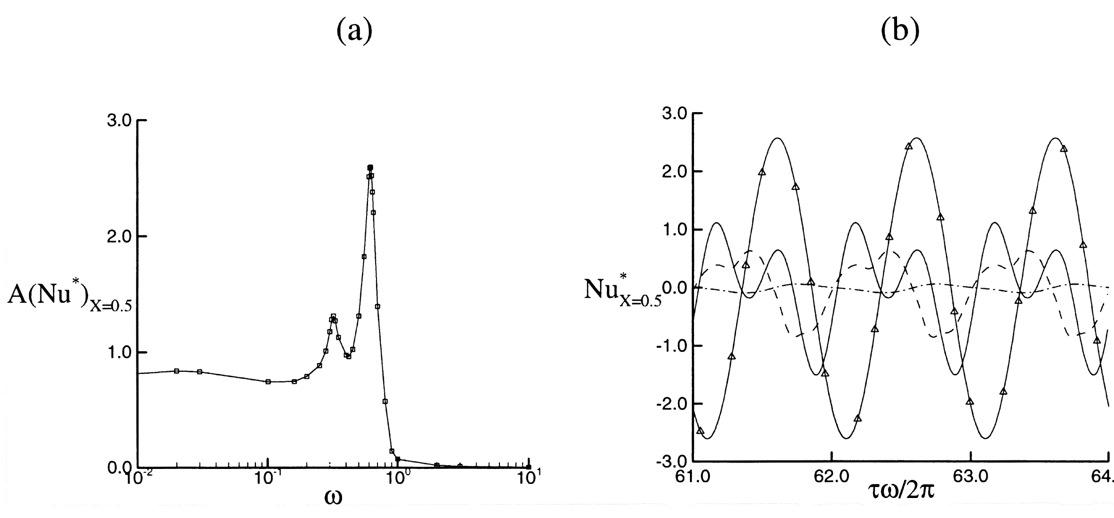


Fig. 3. Behavior of *Nu* at the mid-plane $X = 0.5$. (a) Amplitude of fluctuating part, $A(Nu^*)$, versus ω, (b) time history of Nu^*. (- - -), $\omega = 0.1$; (———), $\omega = 0.32$; $\triangle$, $\omega = 0.62$; (– · – · –), $\omega = 1.0$.

Acknowledgements

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References

[1] J.L. Lage, A. Bejan, The resonance of natural convection in an enclosure heated periodically form the side, Int. J. Heat Mass Transfer 36 (1993) 2027–2038.

[2] B.V. Antohe, J.L. Lage, A dynamic thermal insulator: inducing resonance within a fluid saturate porous medium enclosure heated periodically from the side, Int. J. Heat Mass Transfer 37 (1994) 771–782.

[3] B.V. Antohe, J.L. Lage, Experimental investigation on pulsating horizontal heating of a water-filled enclosure, J. Heat Transfer 118 (1996) 889–896.

[4] B.V. Antohe, J.L. Lage, Amplitude effect on convection induced by time-periodic horizontal heating, Int. J. Heat Mass Transfer 39 (1996) 1121–1133.

[5] H.S. Kwak, J.M. Hyun, Natural convection in an enclosure having a vertical sidewall with time-varying temperature, J. Fluid Mech 329 (1996) 65–88.

[6] H.S. Kwak, K. Kuwahara, J.M. Hyun, Resonant enhancement of natural convection heat transfer in a square enclosure, Int. J. Heat Mass Transfer 41 (1998) 2837–2846.

[7] H.S. Kwak, K. Kuwahara, J.M. Hyun, Prediction of the resonance frequency of natural convection in an enclosure with time-periodic heating imposed on one sidewall, Int. J. Heat Mass Transfer 41 (1998) 3157–3160.

[8] D.R. Moore, N.O. Weiss, Nonlinear penetrative convection, J. Fluid Mech 61 (1973) 553–581.

[9] H.S. Kwak, K. Kuwahara, J.M. Hyun, Convective cooldown of a contained fluid through its maximum density temperature, Int. J. Heat Mass Transfer 41 (1998) 323–333.

[10] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill, New York, 1980.

[11] T. Hayase, J.A.C. Humphery, R. Grief, A consistently formulated QUICK scheme for fast and stable convergence using finite-volume iterative calculation procedures, J. Comput. Phys 98 (1992) 108–118.

[12] T. Nishimura, A. Wake, E. Fukumori, Natural convection of water near the density extremum for a wide range of Rayleigh numbers, Numer. Heat Transfer 27 (1995) 443–449.

[13] S. Paolucci, D.R. Chenoweth, Transition to chaos in a differentially heated vertical cavity, J. Fluid Mech 201 (1989) 379–410.